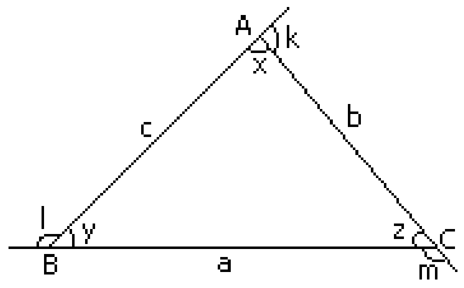
ÜÇGENDE AÇILAR — 1

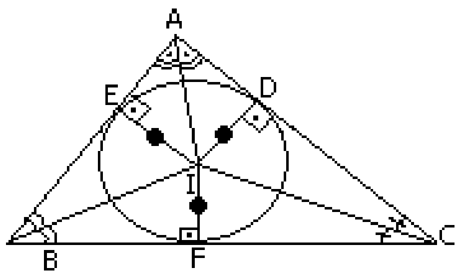


Üçgen Çeşitleri:
1-Kenarlarına göre: Çeşitkenar, İkizkenar ve Eşkenar Üçgen.
2-Açılarına göre: Dar Açılı, Dik Açılı ve Geniş Açılı Üçgen.

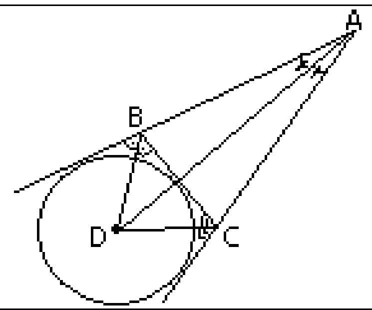
Açı Bağıntıları:
1-Bir köşedeki bir iç açı ile bir dış açı birbirinin bütünleridir.
$x+k=180^0$ $y+l=180^0$ $z+m=180^0$
2- Üçgenin iç açıları ölçüleri toplamı 180^0 dir. $x+y+z=180^0$
3- Üçgenin dış açıları ölçüleri toplamı 360^0 dir. $k+l+m=360^0$
4- İki iç açının toplamı, kendilerine komşu olmayan dış açının toplamına eşittir. $x+z=l$ $x+y=m$ $y+z=k$

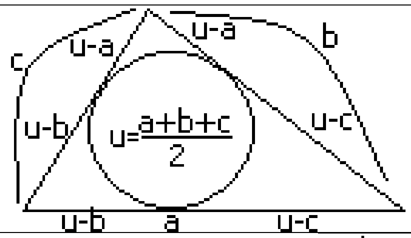
ÜÇGENDE KESİŞEN DOĞRULAR

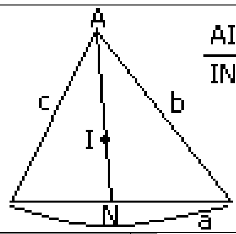
AÇIORTAY



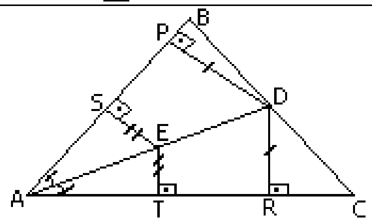
Üçgende iç açıortaylar, çemberin iç teğet çember merkezinde (I) kesişirler. Çember üçgeni D, E ve F noktalarında keser.
[ID]=[IE]=[IF] Bu doğru parçaları çemberin yarıçapıdır ve birbirine eşittir.

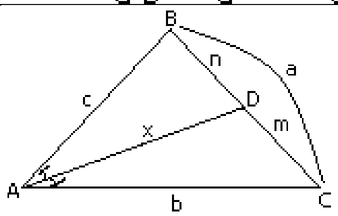




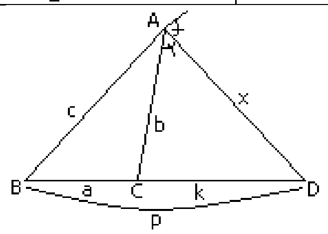


$$\frac{AI}{IN}=\frac{b+c}{a}$$



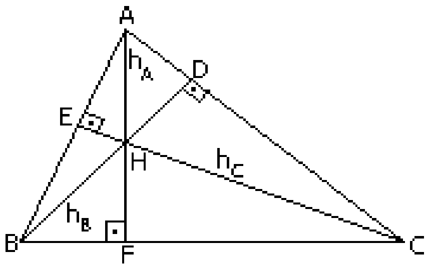


İç açıortayın uzunluğu
$x=|AD|=\sqrt{b.c-m.n}$
bağıntısı ile bulunur.
Ayrıca
$\frac{b}{m}=\frac{c}{n}$ bağıntısı vardır



Dış açıortayın uzunluğu
$x=|AD|=\sqrt{k.p-b.c}$
bağıntısı ile bulunur.
Ayrıca
$\frac{b}{k}=\frac{c}{p}$ bağıntısı vardır

YÜKSEKLİK

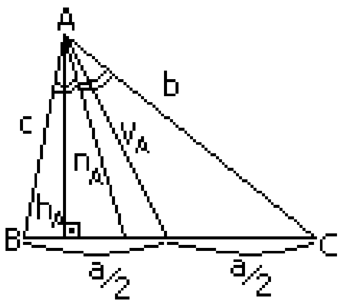


Yüksekliklerin kesiştiği yer (H) üçgenin diklik merkezidir.
Diklik merkezi ile yükseklikler arasında aşağıdaki bağıntı geçerlidir:

$|AH|.|HF|=|BH|.|HD|=|CH|.|HE|$

$h_A+h_B+h_C < |AB|+|BC|+|AC|$

$|AH|^2+|BC|^2=|BH|^2+|AC|^2=|CH|^2+|AB|^2$

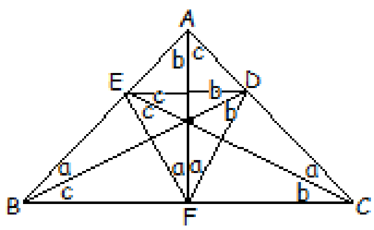


B ve C dar açı ve b>c ise

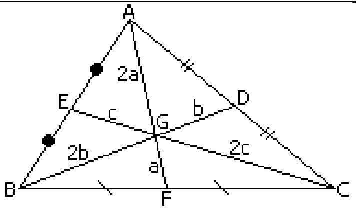
$h_A<n_A<V_A$

$a>b>c \Rightarrow$ $h_A<h_B<h_C$
$n_A<n_B<n_C$
$V_A<V_B<V_C$

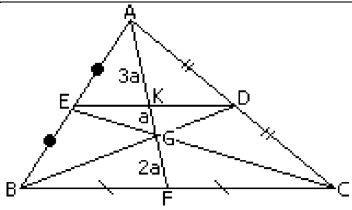
YUSUF KOCAMAN



ABC üçgeninin diklik merkezi, DEF üçgeninin iç teğet çember merkezidir.

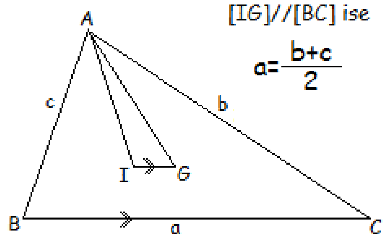


Kenarortaylar üçgenin ağırlık merkezinde kesişirler. Ağırlık merkezi kenarortayları ½ oranında keser.
[AG]=2[GF],
[BG]=2[GD],
[CG]=2[GD].

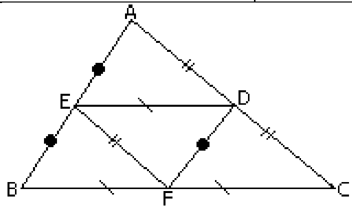


[IG]//[BC] ise

$a = \frac{b+c}{2}$

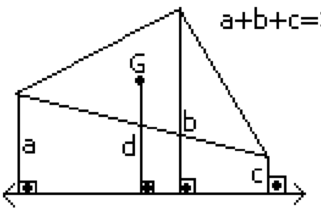


I: içteğet çember mrk
G: ağırlık merkezi

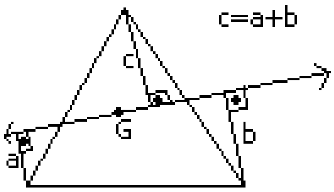


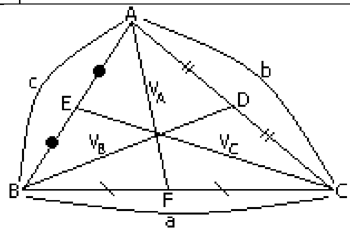
Kenarların orta noktalarını birleştiren doğru parçaları diğer kenara paralel olurlar ve uzunlukları o kenarın yarısı kadar olur.
[ED]//[BC], |BC|=2|ED|
[EF]//[AC], |AC|=2|EF|
[DF]//[AB], |AB|=2|DF|

$a+b+c=3d$



$c=a+b$





$V_A^2 + V_B^2 + V_C^2 = \frac{3}{4}(a^2+b^2+c^2)$

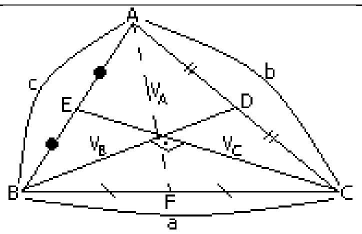
$2V_A^2 = b^2+c^2 - \frac{a^2}{2}$,

$2V_B^2 = a^2+c^2 - \frac{b^2}{2}$

$2V_C^2 = a^2+b^2 - \frac{c^2}{2}$,

$\frac{a+b+c}{2} < V_A+V_B+V_C < a+b+c$

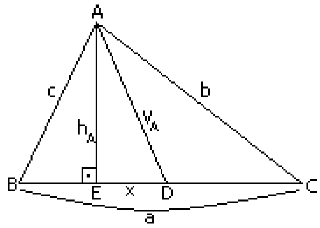
$b+c-a < 2V_A < b+c$



İki kenarortay dik kesişiyorsa ($V_B \perp V_C$ ise)

$V_A^2 = V_B^2 + V_C^2$

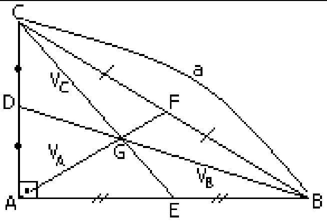
$5a^2 = b^2 + c^2$



Yükseklik ile kenarortay arasında kalan doğru parçasının uzunluğu için ([ED]=x);

$|b^2 - c^2| = 2ax$

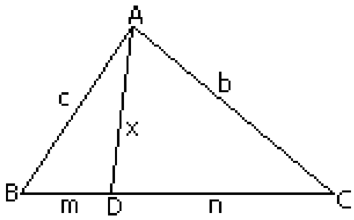
bağıntısı geçerlidir.



Dik üçgende

$5V_A^2 = \frac{5a^2}{4} = V_B^2 + V_C^2$

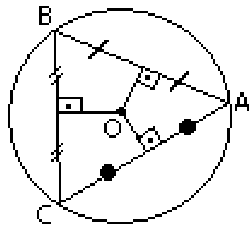
Stewart Bağıntısı:



$X^2 = \frac{b^2.m + c^2.n}{m+n} - m.n$

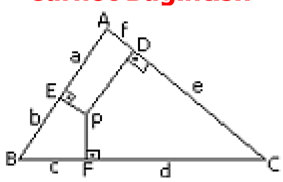
$b=c$ ise $x^2=b^2 - m.n$

Kenar Orta Dikmesi:



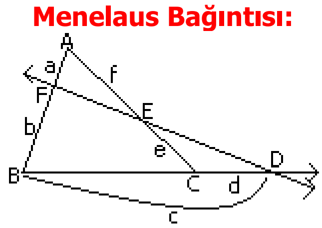
Üçgenin kenarlarının orta noktalarından çıkan dikmeler, üçgenin köşelerinden geçen çevrel çemberinin merkezinde kesişirler.

Carnot Bağıntısı:



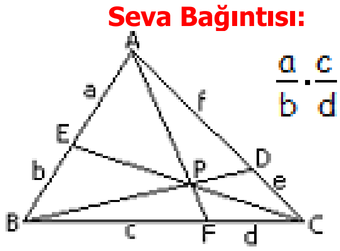
$a^2+c^2+e^2=b^2+d^2+f^2$

Menelaus Bağıntısı:



$\frac{a}{b} \cdot \frac{c}{d} \cdot \frac{e}{f} = 1$

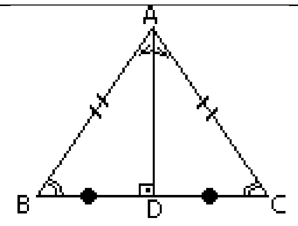
Seva Bağıntısı:

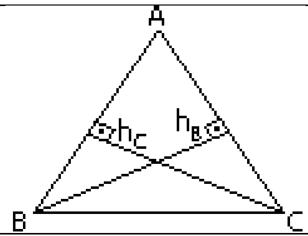


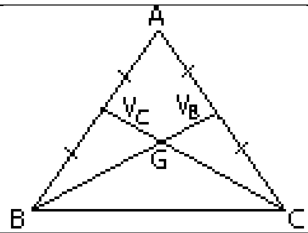
$\frac{a}{b} \cdot \frac{c}{d} \cdot \frac{e}{f} = 1$

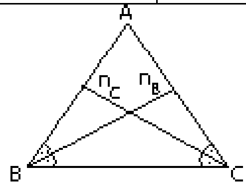
YUSUF KOCAMAN

İKİZKENAR ÜÇGEN







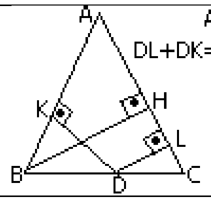


Bir üçgende |AB|=|AC| ise bu üçgene ikizkenar üçgen denir. A 'daki açıya tepe açısı, B ve C deki açılara taban açıları denir ve taban açıları birbirine eşittir.

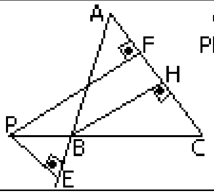
$$m(\widehat{ABC})=m(\widehat{ACB})$$

Ayrıca tepe noktasından inen yükseklik aynı zamanda açıortay ve kenarortaydır. ($h_A=n_A=V_A$)

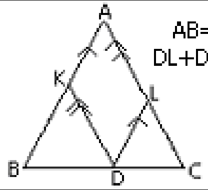
|AB|=|AC| ise $h_B=h_C$,
$V_B=V_C$,
$n_B=n_C$.



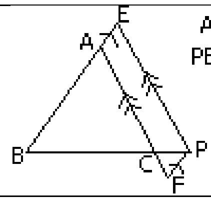
AB=AC
$DL+DK=BH=h_a=h_b$



AB=AC
PF - PE=BH

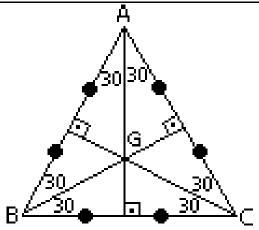


AB=AC ise
DL+DK=AC=AB



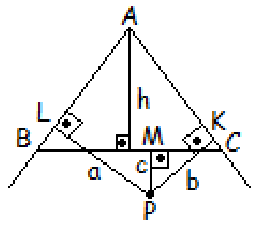
AB=AC ise
PE - PF = AB

EŞKENAR ÜÇGEN



Kenar uzunlukları birbirine eşit olan üçgene eşkenar üçgen denir. İç açıları 60^0 dir.

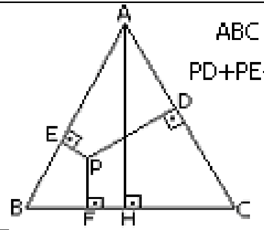
$h_A=h_B=h_C=n_A=n_B=n_C=V_A=V_B=V_C$



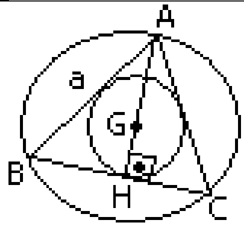
a + b – c = h

veya

|PL| + |PC| - |PM| = h



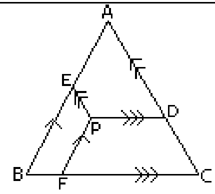
ABC eşkenar üçgen
$PD+PE+PF=h_A=h_B=h_C$



$$GA=R=\frac{a\sqrt{3}}{3}$$

$$GH=r=\frac{a\sqrt{3}}{6}$$

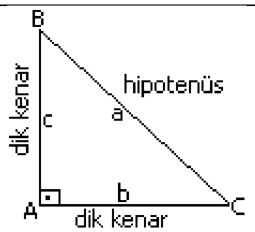
R=2r



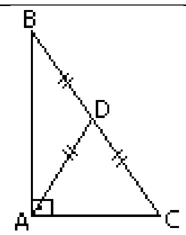
Eşkenar üçgende P herhangi bir nokta ve [PD]//[BC], [PE]//[AC] [PF]//[AB] ise;

|PD|+|PE|+|PF|=|AB|

DİK ÜÇGEN

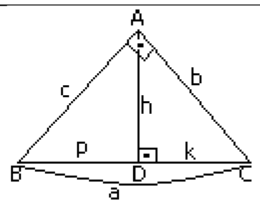


Pisagor bağıntısı:
$a^2=b^2+c^2$



muhteşem üçlü
|AD|=|BD|=|DC|

Dik kenarlar	-	hipotenüs
3 - 4	-	5
5 – 12	-	13
8 – 15	-	17
7 – 24	-	25
9 – 40	-	41
11 – 60	-	61
20 – 21	-	29

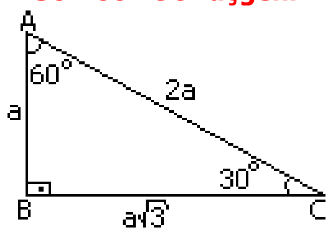


Bir dik üçgende hipotenüse ait yükseklik varsa **öklit bağıntıları** geçerlidir.

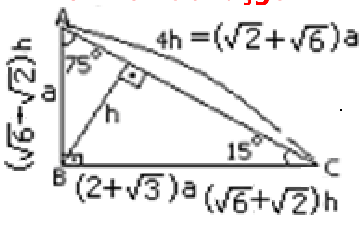
$h^2=p.k$ $b^2=k.a$ $c^2=p.a$

$a.h=b.c$ $\frac{1}{h^2}=\frac{1}{b^2}+\frac{1}{c^2}$

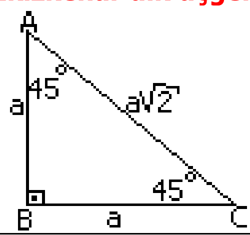
30^0 60^0 90^0 üçgeni

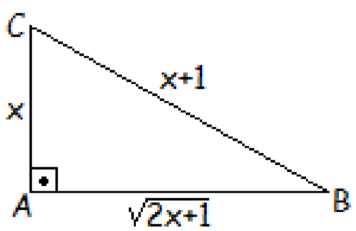


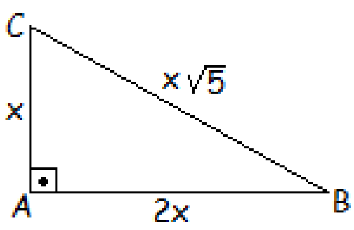
15^0 75^0 90^0 üçgeni

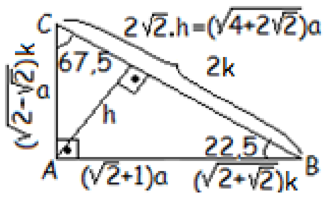


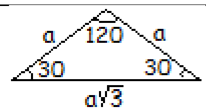
İkizkenar dik üçgen





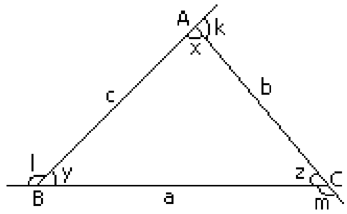






YUSUF KOCAMAN

AÇI KENAR BAĞINTILARI

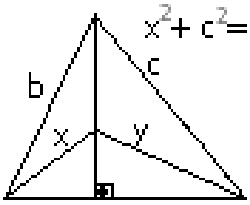


Bir üçgenin iki açısı eş değilse, bunların karşısındaki kenarlarda eş değildir. Daha büyük olan açının karşısındaki kenar daha uzundur.

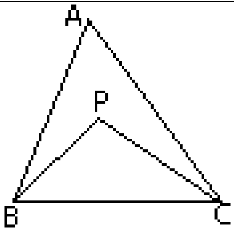
a<b<c ⇔ x<y<z ⇔ k>l>m

$|b-c|<a<b+c$

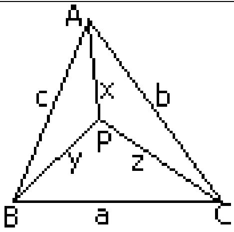
$x=90^0$	ise	$a^2=b^2+c^2$
$x<90^0$	ise	$\lvert b-c\rvert < a < \sqrt{b^2+c^2}$
$x>90^0$	ise	$b+c > a > \sqrt{b^2+c^2}$



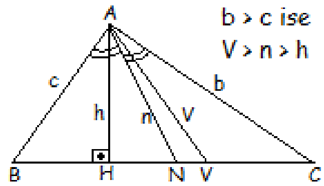
$x^2+c^2=y^2+b^2$



$|BC| < |PB|+|PC| < |AB|+|AC|$

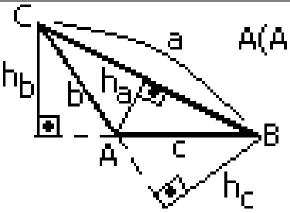


$\frac{a+b+c}{2} < x+y+z < a+b+c$



b > c ise
V > n > h

ÜÇGENDE ALAN



$A(ABC)=\frac{a.h_a}{2}$

$\frac{b.h_b}{2}=\frac{c.h_c}{2}$

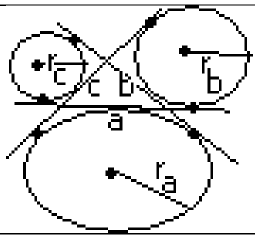
$u=\frac{a+b+c}{2}$

$A(ABC)=\sqrt{u.(u-a).(u-b).(u-c)}$

$=\frac{1}{2}.b.c.\sin A$

Alan=u.r

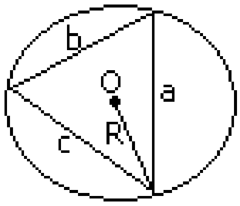
$=\sqrt{r.r_a.r_b.r_c}$



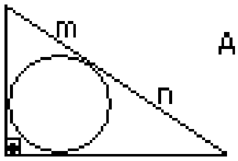
Alan= $u.(r_a-a)$

$u.(r_b-b)$

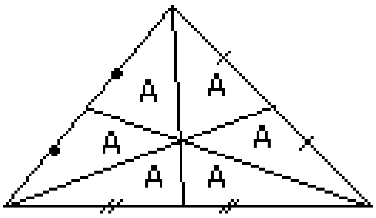
$u.(r_c-c)$

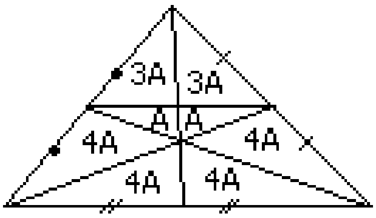


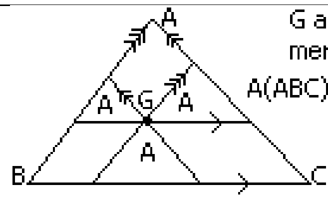
$Alan=\frac{a.b.c}{4R}$



Alan=m.n

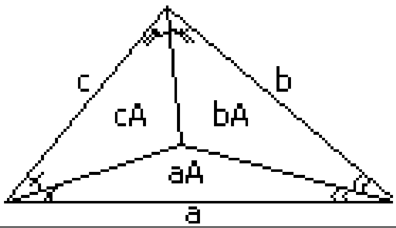


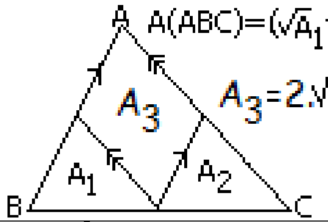




G ağırlık merkezi

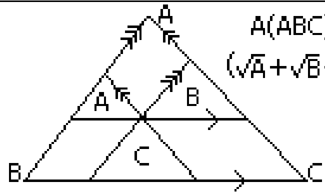
A(ABC)=9A





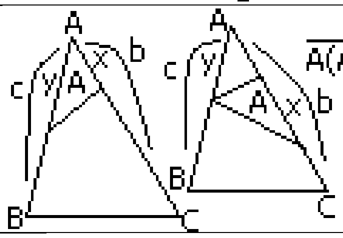
$A(ABC)=(\sqrt{A_1}+\sqrt{A_2})^2$

$A_3=2.\sqrt{A_1.A_2}$

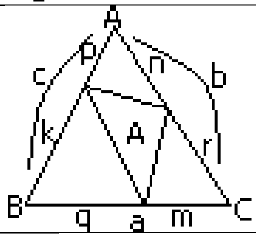


A(ABC)=

$(\sqrt{A}+\sqrt{B}+\sqrt{C})^2$

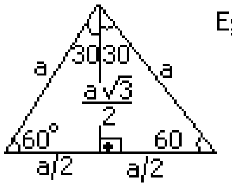


$\frac{A}{A(ABC)}=\frac{x.y}{b.c}$



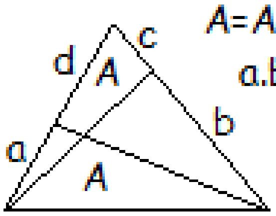
$\frac{A}{A(ABC)}=$

$\frac{p.q.r+k.m.n}{a.b.c}$



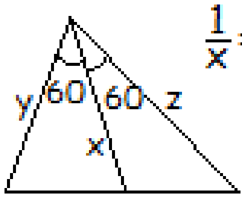
Eşkenar üçgen

$Alan=\frac{a^2\sqrt{3}}{4}$

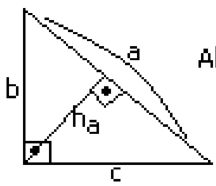


A=A ise

a.b=c.d



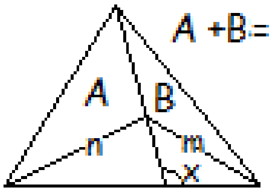
$\frac{1}{x}=\frac{1}{y}+\frac{1}{z}$



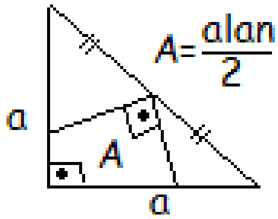
Dik üçgen

$Alan=\frac{b.c}{2}=\frac{a.h_a}{2}$

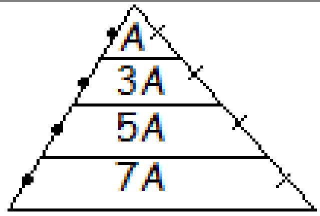
YUSUF KOCAMAN



$$A + B = \frac{m.n.\sin x}{2}$$



$$A = \frac{alan}{2} = \frac{a^2}{2}$$

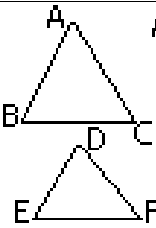


a,b,c nin üçü de ayrı ayrı biliniyorsa S alan ise

$$\sqrt{\frac{a^2+b^2+c^2+4\sqrt{3}.S}{2}} \leq x+y+z < \text{en uzun iki kenar toplamı}$$

YUSUF KOCAMAN

BENZERLİK

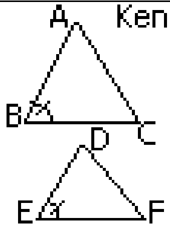


ABC ∼ DEF ise

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = k$$

benzerlik oranı

Kenar Açı Kenar

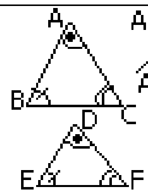


$\frac{AB}{DE} = \frac{BC}{EF} = k$ ise

ABC ∼ DEF dir.

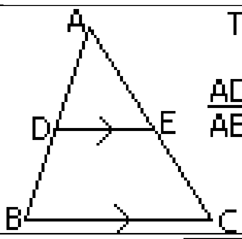
$$k = \frac{AC}{DF}$$

Açı Açı Açı



$\hat{A}=\hat{D}$, $\hat{B}=\hat{E}$, $\hat{C}=\hat{F}$ ise ABC ∼ DEF dir.

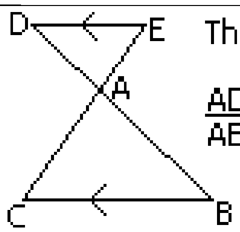
Temel Orantı



$$\frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC} = k$$

$$\frac{AD}{DB} = \frac{AE}{EC}$$

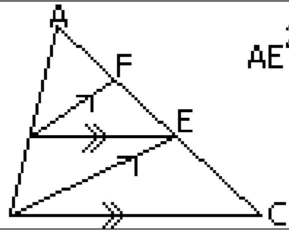
Thales (Kelebek)



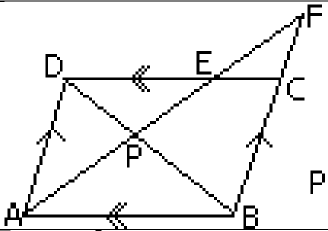
$$\frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC} = k$$

$$\frac{a}{d} = \frac{b}{e} = \frac{c}{f} = \frac{h_a}{h_e} = \frac{h_b}{h_f} = \frac{h_c}{h_d} = \frac{n_A}{n_D} = \frac{n_B}{n_E} = \frac{n_C}{n_F} =$$

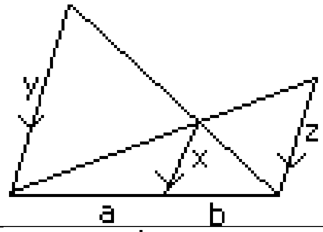
$\frac{V_a}{V_d} = \frac{V_b}{V_e} = \frac{V_c}{V_f} = k$ iç, dış teğet, çevrel çember yarıçapları oranı da k dır.



$$AE^2 = AF.AC$$

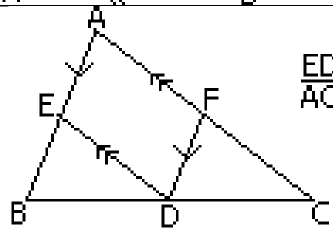


$$PA^2 = PE.PF$$

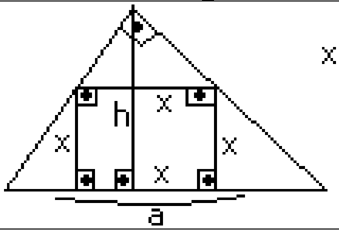


$$\frac{1}{x} = \frac{1}{y} + \frac{1}{z}$$

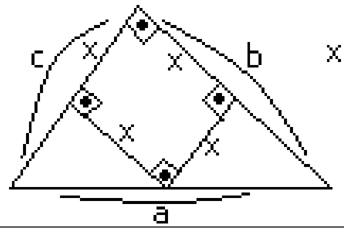
$$a.z = b.y$$



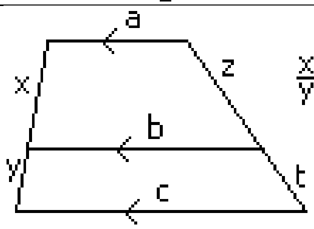
$$\frac{ED}{AC} + \frac{DF}{AB} = 1$$



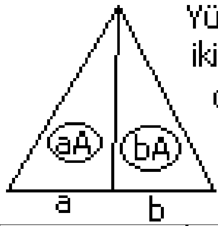
$$x = \frac{a.h}{a+h}$$



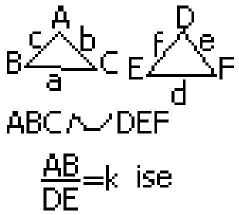
$$x = \frac{b.c}{b+c}$$



$$\frac{x}{y} = \frac{z}{t} = \frac{b-a}{c-b}$$



Yükseklikleri eş olan iki üçgenin alanları oranı tabanlarına eşittir

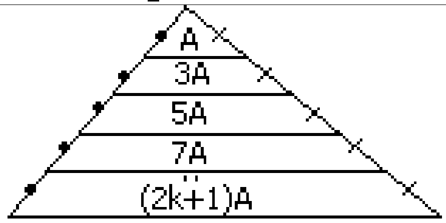


ABC ∼ DEF

$\frac{AB}{DE} = k$ ise

$$\frac{A(ABC)}{A(DEF)} = k^2$$

$$\frac{Ç(ABC)}{Ç(DEF)} = k$$

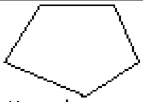


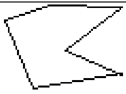
ÇOKGENLER

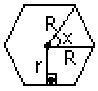
n kenarlı konveks çokgenin
- iç açıları toplamı $(n-2).180^\circ$
- dış açıları toplamı 360°
- köşegen sayısı $\frac{n.(n-3)}{2}$

n kenarlı konveks çokgenin
- bir köşesinden $(n-3)$ tane köşegen çizilebilir.
- bu köşegenler $(n-2)$ tane üçgensel bölge oluşturur.

n kenarlı konveks çokgenin
- çizilebilmesi için $(2n-3)$ eleman gereklidir.
  - en az $(n-2)$ tanesi uzunluk
  - en çok $(n-1)$ tanesi açı


Konveks (dışbükey)

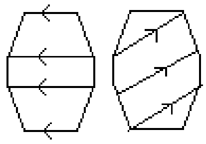

Konkav (içbükey)



n kenarlı düzgün çokgen

u=Çevre/2

Alan= $u.r=\frac{1}{2}(R^2\sin x).n$

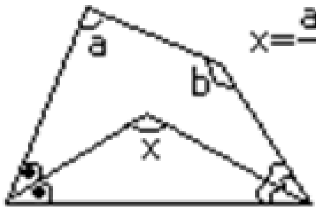


Düzgün Çokgende

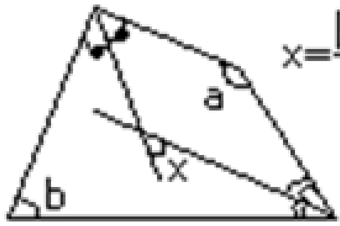
Düzgün Altıgenin bir kenar uzunluğu a ise

Alanı= $6\frac{a^2\sqrt{3}}{4}$

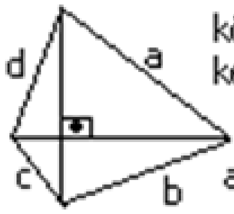
DÖRTGENLER



$x=\frac{a+b}{2}$

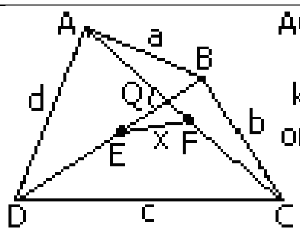


$x=\frac{|a-b|}{2}$



köşegenler dik kesişiyorsa

$a^2+c^2=b^2+d^2$



AC=e, BD=f

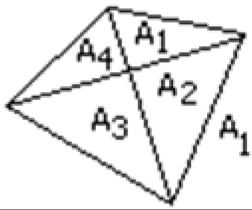
E,F köşegenlerin orta noktaları ise ⟹

$e^2+f^2=a^2+b^2+c^2+d^2-4x^2$

$u=\frac{a+b+c+d}{2}$

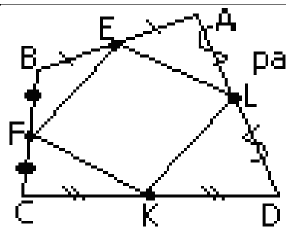
$u<AC+BD<2u$

$Alan=\frac{e.f.\sin Q}{2}$



Alanlar belirtildiği gibi ise

$A_1.A_3=A_2.A_4$



EFKL paralelkenardır. ⟹

Çevre(EFKL)=AC+BD

$Alan(EFKL)=\frac{Alan(ABCD)}{2}$

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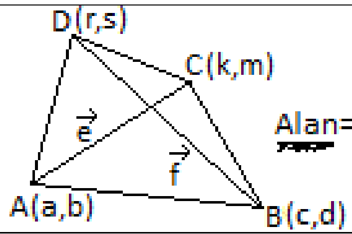


$Alan=\frac{1}{2}\sqrt{|\vec{e}|^2.|\vec{f}|^2-<\vec{e},\vec{f}>^2}$

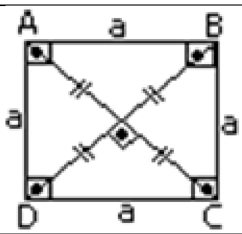
$\vec{e}=(x\ ,\ y)=(a-k\ ,\ b-m)$

$\vec{f}=(z\ ,\ t)=(c-r\ ,\ d-s)$

$|\vec{e}|^2=(x)^2+(y)^2$ $\quad <\vec{e},\vec{f}>=x.z+y.t$

$|\vec{f}|^2=(z)^2+(t)^2$

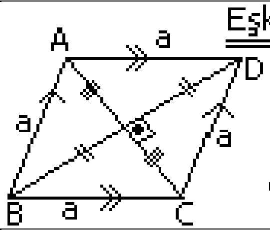
KARE - EŞKENAR DÖRTGEN – DELTOİD – DİKDÖRTGEN



Kare

Çevre=4a

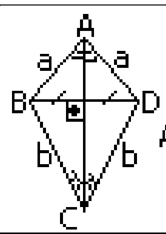
$Alan=a^2=\frac{AC^2}{2}$



Eşkenar Dörtgen

Çevre=4a

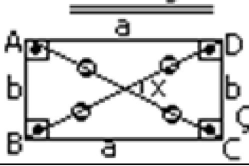
$Alan=\frac{AC.BD^2}{2}$



Deltoid

$Alan=\frac{AC.BD}{2}$

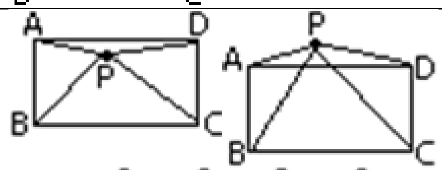
Dikdörtgen



Alan= a.b

$=\frac{1}{2}AC^2\sin x$

Çevre=2a+2b



$AP^2+PC^2=BP^2+PD^2$

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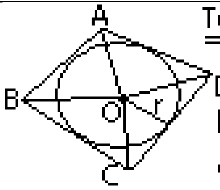
KİRİŞLER DÖRTGENİ - TEĞETLER DÖRTGENİ

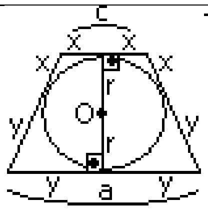


Teğetler Dörtgeni

Merkez açıortayların kesim noktasıdır.

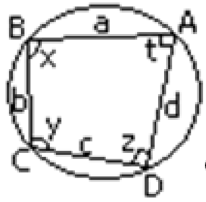
$AB+CD=BC+AD$

$Alan=u.r$



Teğetler dörtgeni ikizkenar yamuk ise

$h=2.r=\sqrt{a.c}$

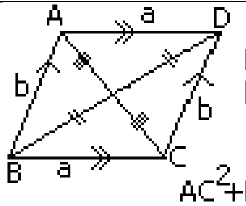


Kirişler Dörtgeni

$x+z=y+t=180^\circ$

$a.c+b.d=BD.AC$

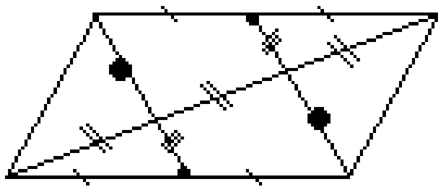
PARALELKENAR

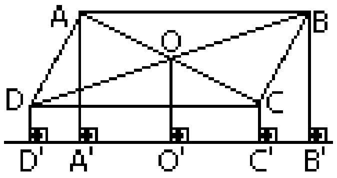


$A=C$ $B=D$

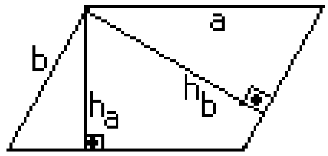
Komşu açılar bütünlerdir.

$AC^2+BD^2=2(a^2+b^2)$

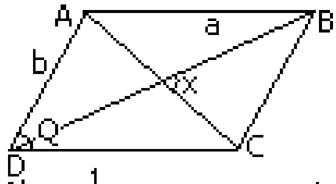




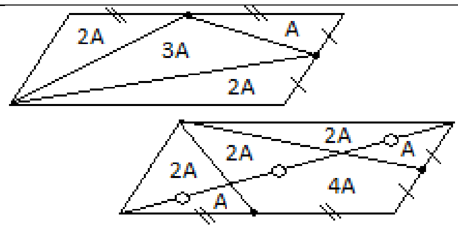
$DD'+BB'=AA'+CC'=2.OO'$

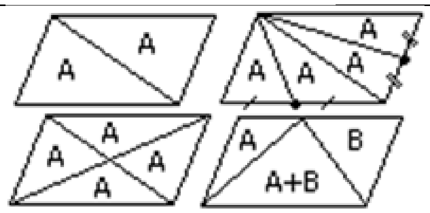


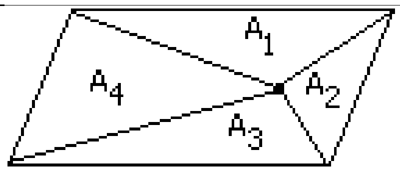
$Alan=a.h_a=b.h_b$



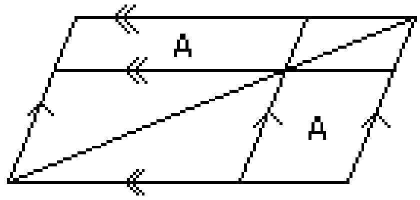
$Alan=\frac{1}{2}AC.BD.sinx=a.b.sinQ$



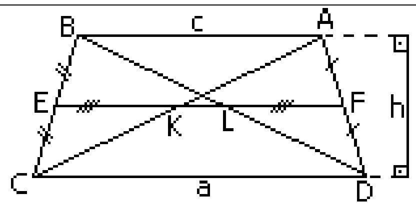




$A_1+A_3=A_2+A_4$



YAMUK



AB:üst taban

CD:alt taban

EF:orta taban

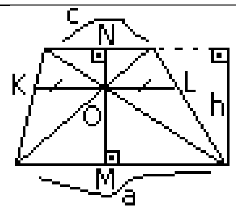
AB//CD//EF

$EK=LF=\frac{c}{2}$

$A+D=B+C=180^\circ$

$EF=\frac{a+c}{2}$ $KL=\frac{|a-c|}{2}$

$Alan=\frac{(a+c).h}{2}$

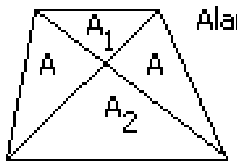


$OK=a.c/(a+c)$

$OM=a.h/(a+c)$

$ON=c.h/(a+c)$

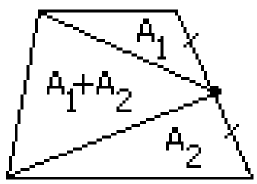
$OM/ON=a/c$

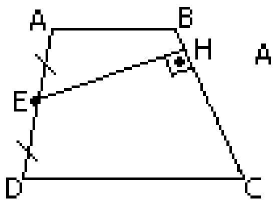


$Alan=(\sqrt{A_1}+\sqrt{A_2})^2$

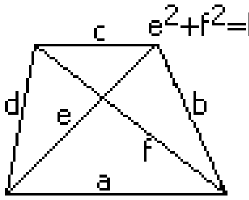
$A_1.A_2=A^2$

YUSUF KOCAMAN

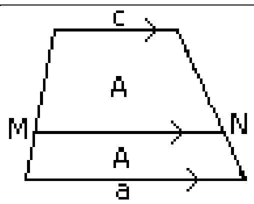




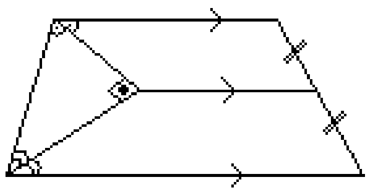
$Alan=BC.EH$



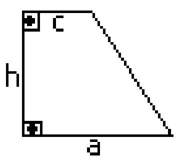
$e^2+f^2=b^2+d^2+2.a.c$

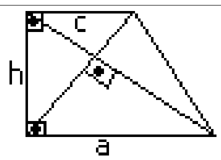


$MN=\sqrt{\frac{a^2+c^2}{2}}$



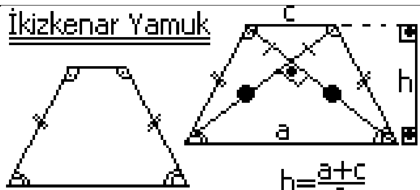
Dik Yamuk



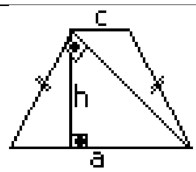


$h^2=a.c$

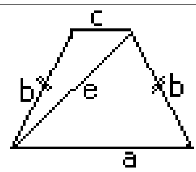
İkizkenar Yamuk



$h=\frac{a+c}{2}$



$4h^2=a^2-c^2$



$e^2=b^2+a.c$

YUSUF KOCAMAN